Methods of Determining Internal Parameters of Varistor Model

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***Abstract*- ZnO varistors are semiconductor devices with highly nonlinear current-voltage characteristic and are widely used as devices for overvoltage protection. Varistor applications range from the use of small varistors to protect electronic components to large varistors for protection of power systems. This paper presents proposed model of ZnO varistor and methodology of its mathematical analysis and simulation. The mathematical analy-sis of the proposed model makes it possible simulate the current trace on a nonlinear element.**

I. INTRODUCTION

Varistors are ceramic elements whose current-voltage (I-V) characteristic is highly nonlinear [1]. The varistors are usually manufactured in the ceramic process, in which pressed zinc oxide with admixtures of other metallic oxides is sintered. A matrix made up of ZnO grains enclosed by an intergranular layer composed of dissolved oxide admixtures forms the obtained microstructure [2-5].

Numerous, more or less complicated ZnO varistor models can be found in the literature [6-10]. The above model is a starting point for our considerations, but under the assumption that the influence of the capacitance nonlinearity is negligible and that the additional impedance due to the finite conduc-tance and capacitance of the intergranular phase has a signifi-cant effect on the model response.

Thus the simple ZnO varistor model proposed here can be presented as:

- the nonlinear resistance of intergranular boundaries and the linear capacitance associated with the impoverished re-gion, and

- the capacitance-resistance impedance associated with the intergranular phase. Your goal is to simulate the usual appearance of papers in an *IEEE conference proceedings*. For items not addressed in these instructions, please refer to the last issue of your conference's proceedings or your Publications chair.

II. MATHEMATICAL ANALYSIS OF THE MODEL

The analysed ZnO varistor model is shown in Fig. 1. The superposition method, the first harmonic method or the con-volution method can used to describe mathematically the equivalent circuit diagram, assuming that the supply voltage is sinusoidal.

The superposition method was used to describe the varistor model in [11].

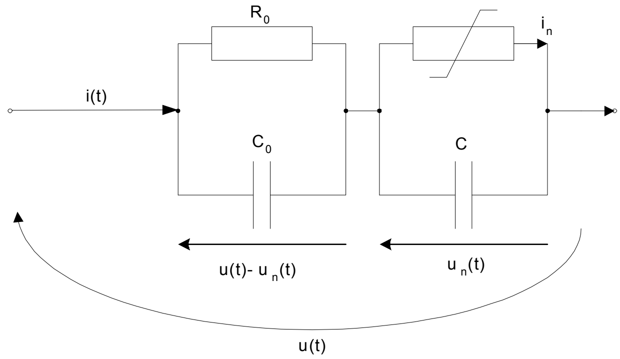

Figure 1. Analytical model for the ZnO varistor

If the model's parameters are known, it can be described using the first harmonic method.

A. First harmonic method

The relationship between the rms values of u(t) and (t) is as follows:

$$U = F(I) \tag{1}$$

The rms values are assumed to be first harmonic rms values:

$\underline{U}_{(\omega)} = Ue^{j\varphi}$; $\underline{I}_{(\omega)} = I$ and $\underline{U}_{(k\omega)} = 0$; $\underline{I}_{(k\omega)} = 0$ for k>1

Hence

$$\underline{U}_{N(\omega)} = Ue^{j\varphi} - \underline{Z}_{A(\omega)} I \tag{2}$$

$$\underline{I}_{N(\omega)} = [1 + \underline{Y}_{B(\omega)} \underline{Z}_{A(\omega)}] I - \underline{Y}_{B(\omega)} Ue^{j\varphi} \tag{3}$$

Such a phase shift φ should be selected that $\underline{U}_{N(\omega)}$ and $\underline{I}_{N(\omega)}$ are in phase (as on a nonlinear resistance element).

Thus φ must satisfy this equation:

$$\mathrm{Re}[\underline{U}_{N(\omega)}]\,\mathrm{Im}[\underline{I}_{N(\omega)}] = \mathrm{Im}[\underline{U}_{N(\omega)}]\,\mathrm{Re}[\underline{I}_{N(\omega)}] \tag{4}$$

Equation (1) is calculated for an assumed value of I and from the equations:

$U_N = \left|\underline{U}_{N(\omega)}\right|$, $I_N = \left|\underline{I}_{N(\omega)}\right|$ for a determined value of φ.

In this way the relationship which describes nonlinear element $U_N = f_I(I_N)$ is determined parametrically since U_N and I_N are parameterised by variable I.

It should be noted that the nonlinear element is treated as quasilinear, i.e. linear for transients and nonlinear for rms values, and that in the above methods the varistor's internal parameters are assumed to be known and their values affect the form of the solution.

B. *Calculation of internal parameters*

The varistor model consisting of linear elements with unknown parameters R_0, C_0, C and a resistance element with unknown characteristic $u_n = f(i_n)$.

The circuit's current response is described by this equation:

$$i(t) = \frac{1}{R_0}[u(t) - u_n(t)] + C_0 \frac{d}{dt}[u(t) - u_n(t)] \tag{5}$$

where:

$u(t) = U_m \sin \omega t$ - the circuit supply voltage;

$u_n(t) = U_{1n} \sin(\omega t + \Psi_1) + U_{3n} \sin(3\omega t + \Psi_3)$ - the voltage on the nonlinear element, consisting (for simplicity) of the first harmonic and the second harmonic;

R_0 – an unknown resistance of the model's linear element;

C_0 – unknown capacitances of the model's linear element;

$U_{1n}, U_{3n}, \Psi_1, \Psi_3$ - respectively unknown amplitudes and initial phases for the first harmonic and the second harmonic of the model nonlinear element voltage.

For the selected 6 times (as many as there are unknowns) and the corresponding known values of voltage samples and the varistor model supply voltage the following system of equations:

$$i(t_l) = \frac{1}{R_0}[u(t_l) - u_n(t_l)] + C_0 \{\frac{d}{dt}[u(t) - u_n(t)]\}_{t=t_l} \tag{6}$$

where:

t_l – for l=1,2,...,6 is solved.

Having calculated R_0, C_0 and the nonlinear element voltage, one can determine capacitance C. For this purpose we formulate the following equation for the varistor nonlinear element voltage:

$$f(u_n) = i(t) - C\left(\frac{du_n}{dt}\right) \tag{7}$$

For suitably selected t_0 at which: $u_n(t_0) = 0$ equation (7) assumes this form:

$$f(u_n(t_0)) = i(t_0) - C\left(\frac{du_n}{dt}\right)_{t_0} = 0\,, \tag{8}$$

whereby it becomes possible to calculate linear capacitance C of varistor model.

III. Conclusions

Two more mathematical descriptions of the varistor model have been presented. They can be applied to solve problems connected with the operation of varistors as nonlinear elements in electric circuits. The presented *first harmonic method* can be treated as an alternative to the superposition method described in [11] if the model parameters are known. If not, one can determine them in the way described in this paper's section IIB.

Currently intensive research on the application of the above problems to the analysis of actual varistor operation is being conducted.

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